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$$p=\sqrt{(HO_1.O_1K)}=\sqrt{(AD.AF)}\sin\beta, \qquad q=\sqrt{(ZO_2.O_2I)}=\sqrt{(AB.AR)}\sin\beta,$$

$$AD-AF=FE=\frac{DF\sin\theta}{\cos\beta}, \qquad AB-AR=RC=\frac{BR\sin\phi}{\cos\beta}.$$

$$\therefore \frac{GX}{O_1O_2}=\frac{6(p^3DF\sin\theta-q^3BR\sin\phi)}{8(BR\sin\phi-DF\sin\theta)(q^3-p^3)}.$$

Let $DF=2P$, $BR=2Q$. Then

$$p=\frac{P\sqrt{[\cos(\theta+\beta)\cos(\theta-\beta)]}}{\cos\beta}=\frac{P\sqrt{[\cos^2\theta-\sin^2\beta]}}{\cos\beta}, \quad q=\frac{Q\sqrt{[\cos^2\theta-\sin^2\beta]}}{\cos\beta}.$$

$$\therefore \frac{GX}{O_1O_2}=\frac{6[P^4\sin\theta(\cos^2\theta-\sin^2\beta)^{\frac{3}{2}}-Q^4\sin\phi(\cos^2\phi-\sin^2\beta)^{\frac{3}{2}}]}{8\{[Q\sin\phi-P\sin\theta][Q^3(\cos^2\phi-\sin^2\beta)^{\frac{3}{2}}-P^3(\cos^2\theta-\sin^2\beta)^{\frac{3}{2}}]\}}.$$

If the planes of the ellipses are parallel, $\theta=\phi$, and we get

$$\frac{GX}{O_1O_2}=\frac{6(P^4-Q^4)}{8(Q-P)(Q^3-P^3)}=\frac{6(P^2+Q^2)(P+Q)}{P^3-Q^3}.$$

MISCELLANEOUS.

165. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that $\tan^{-1}\frac{n}{n+1}+\tan^{-1}\frac{1}{2n+1}=\frac{1}{4}\pi$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.; A. H. HOLMES, Brunswick, Me.; J. EDWARD SANDERS, Reinersville, O.; FRANK M. DRYZER, A. B., Knoxville, Tenn.; and PROF. J. W. NICHOLSON, State University, La.

Let $\alpha=\tan^{-1}\frac{n}{n+1}$ and $\beta=\tan^{-1}\frac{1}{2n+1}$.

Then $\tan\alpha=\frac{n}{n+1}$, $\tan\beta=\frac{1}{2n+1}$, and $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{n}{n+1}+\frac{1}{2n+1}}{1-\frac{n}{(n+1)(2n+1)}}=\frac{2n^2+2n+1}{2n^2+2n+1}=1.$$

$\therefore \alpha+\beta=\frac{1}{4}\pi$.

Also solved by G. W. Greenwood.

166. Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

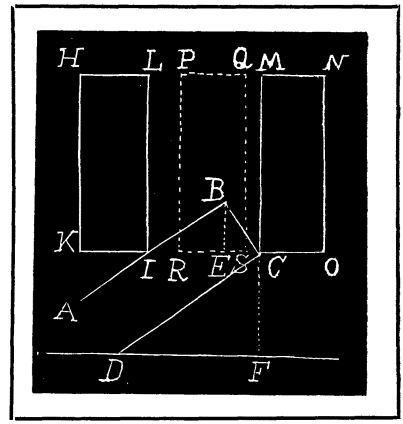
Several equal rectangular boxes are placed in a row with uniform intervals between the boxes and a passageway along one side of the row. Find the least width of the passageway permitting a box to be removed from the row without moving adjacent boxes. This problem arose during the construction of a room for storage batteries.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let *HLIK*, *MNOC* be two of the boxes, with the box *PQSR* occupying the space between them in the position *ABCD*. Let $KH=MC=AB=a$, the length of the box; $KI=CO=BC=b$, the width of the box; $IR=SC=c$, the interval between boxes.

When the box is in the position *ABCD* it can be taken out without removing the adjacent box or boxes.

Then *CF* is the width of the passageway. $CF=CD\sin CDF=a\sin\theta$.



$$IC=\frac{BC}{\cos\theta}=\frac{b}{\cos\theta}=b+2c.$$

$$\therefore\cos\theta=\frac{b}{b+2c}.\qquad\therefore CF=\frac{2a}{b+2c}\sqrt{[c(b+c)]}.$$

$$\therefore\text{Width}=\frac{2a}{b+2c}\sqrt{[c(b+c)]}.$$

If the height of the box is less than the width, and *IC* wide enough to turn the box on its side, then write b=height and $c=\frac{1}{2}(IC-\text{height})$.

In the above $a>b+2c$, otherwise the passageway would be equal to the width of the box plus a few inches for room.

Also solved by A. H. Holmes.

167. Proposed by DR. OSWALD VEBLEN, Princeton University, Princeton, N. J.

If possible, arrange 43 objects, say the numbers 0, 1, 2, ..., in 43 sets of seven each such that every pair of objects lies in one and only one set of seven. It will then be true that two sets of seven have in common one and only one object.

Discussion by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

The following process of building successive sets of seven each leads to the conclusion that the problem is impossible. Let any set of seven be 0, 42, 41, ..., 37. As no two of this set can occur again together, each of these elements must next be combined with six sets of six elements each, these six elements being chosen from the numbers 36, 35, ..., 1. It will be shown that the twelve sets below may be taken as twelve of the desired sets additional to the above.

42, 36, 35, 34, 33, 32, 31	41, 36, 30, 24, 18, 12, 6
42, 30, 29, 28, 27, 26, 25	41, 35, 29, 23, 17, 11, 5
42, 24, 23, 22, 21, 20, 19 (*A*),	41, 34, 28, 22, 16, 10, 4 (*B*).
42, 18, 17, 16, 15, 14, 13	41, 33, 27, 21, 15, 9, 3
42, 12, 11, 10, 9, 8, 7	41, 32, 26, 20, 14, 8, 2
42, 6, 5, 4, 3, 2, 1	41, 31, 25, 19, 13, 7, 1